Point Set Registration with Integrated Scale Estimation

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Abstract: *We present an iterative registration algorithm for aligning two differently scaled 3-D point sets. It extends the popular Iterative Closest Point (ICP) algorithm by estimating a scale factor between the two point sets in every iteration. The presented algorithm is especially useful for the registration of point sets generated by structure-from-motion algorithms, which only reconstruct the 3-D structure of a scene up to scale.*

Like the original ICP algorithm, the presented algorithm requires a rough pre-alignment of the point sets. In order to determine the necessary accuracy of the pre-alignment, we have experimentally evaluated the basin of convergence of the algorithm with respect to the initial rotation, translation, and scale factor between the two point sets.

Keywords: Iterative Closest Point, Scale Estimation

1. INTRODUCTION

Registration of 3-D data, also known as motion and correspondence estimation, has a wide range of applications. It is used for combining range images obtained by scanning a 3-D object from several viewpoints, in order to build a complete 3-D model of the scanned object [1]. Another application is the comparison of scanned faces to a database of 3-D face models for face recognition [2]. As a final example, it is also employed for the registration of 3-D data reconstructed by structure-from-motion algorithms from endoscopic images to computer tomography data [3].

As the Iterative Closest Point (ICP) algorithm is the most popular algorithm for registration of 3-D data, there are many extensions to the basic algorithm introduced independently by Besl and McKay [4] and Chen and Medioni [5]. A comprehensive summary of different extensions and an experimental evaluation focussing on their speed of convergence can be found in [6].

In many applications, the 3-D data used as input for the ICP algorithm contain high levels of noise and outliers. Consequently, improving the robustness of the registration is a primary concern. Approaches applying the least median of squares technique at different levels of the ICP algorithm have been presented in [7] and [8]. A comparison of three robust algorithms, including the two algorithms mentioned above, can be found in [9]. Recently, the Trimmed ICP algorithm was proposed for robust registration [10].

Another active area of research is the problem of finding a rough pre-alignment of the input data, because the ICP algorithm is susceptible to converging to a local optimum if its starting point is too far from the correct registration. A short summary of different techniques for pre-alignment of the input data can be found in [1].

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Structure-from-motion algorithms only reconstruct the 3-D structure of a scene up to scale. Consequently, if at least one of the data sets to be registered is generated by structure-from-motion algorithms, the additional problem of estimating the scale factor between the two data sets arises. In [3], this problem is solved by estimating the scale factor in a pre-alignment step, before applying the ICP algorithm. In this paper, we show how to estimate the scale factor within the ICP algorithm, which results in a more accurate estimate of the scale factor and, consequently, in a more accurate registration. Furthermore, the conceptual simplicity of the integration allows the simultaneous use of a wide range of other extensions to the ICP algorithm. To the best of our knowledge, a solution for simultaneous estimation of rotation, translation, and scale factor within the ICP algorithm has never been proposed before.

In the next section, we shortly present the structure and the mathematical details of the standard ICP algorithm. In Sect. 3, we describe the proposed extension of the ICP algorithm with integrated scale estimation. Finally, the performance of our algorithm is demonstrated in several experiments on point sets generated by structure-from-motion algorithms in Sect. 4.

2. THE ICP ALGORITHM

As we are mainly interested in working with data obtained by structure-from-motion algorithms, we are focussing our discussion of the ICP algorithm on the use of 3-D point sets. Nevertheless, our proposed algorithm also works with other data representations, like range images, line sets, parametric surfaces, or triangle meshes.

In our case, the input data of the ICP algorithm consists of two point sets, the set of data points A and the set of model points B. The solution of the registration problem is then given by the rotation matrix $\boldsymbol{R} \in \mathbb{R}^{3\times3}$ and the translation vector $\boldsymbol{t} \in \mathbb{R}^3$ that best align the data point set and the model point set.

The basic structure of the ICP algorithm is to iteratively perform the following two steps until convergence. In the first step, corresponding point pairs in the data point set and the model point set are determined. In the second step, the motion that best aligns the corresponding point pairs is computed and applied to the data points. As the registration error is reduced in every step, the standard ICP algorithms always converges in finite time [4].

The first step of the ICP algorithm can be divided into several smaller parts. At first, control points have to be selected. The simple strategy of the standard ICP algorithm is to use all data points as control points. Other strategies include selecting a mixture of data points and model points [6] or using a hierarchical point selection scheme in order to increase the computation speed [9].

Then, for each control point from the set of data points, the closest model point is found using nearest neighbor search. Thus, the set of corresponding point pairs

$$C = \{(i, j) \mid \boldsymbol{a}_i \in A \text{ and } \boldsymbol{b}_j \in B \text{ are corresponding points}\}$$

is formed. As this operation is the most time-consuming operation of the ICP algorithm, we propose to use a highly optimized k-D tree nearest neighbor search algorithm for maximum performance [11].

After the corresponding point pairs have been formed, erroneous point pairs can be rejected. When no additional information is available, only the distance of the two points in a pair can be used to discriminate good pairs from outliers. A versatile method for computing the maximum allowable distance is described in [12]. The main idea is to robustly estimate the standard deviation of the distances, and to reject point pairs with a distance greater than a chosen multiple of this standard deviation.

Although the described outlier rejection is not part of the standard ICP algorithm, it can be applied whenever increased robustness to noise and outliers is required. As a downside, it entails a small chance that the ICP algorithm fails to converge in finite time, which can be handled by setting a threshold for the maximum number of iterations. Another disadvantage is that any kind of outlier rejection reduces the speed of convergence of the ICP algorithm.

The second step of the ICP algorithm is the computation of the motion that aligns the corresponding point pairs, which is also known as the absolute orientation problem. As we use the sum of squared distances of the corresponding point pairs as error measure, the minimization problem can be written as

$$(\boldsymbol{R}^*, \boldsymbol{t}^*) = \underset{\boldsymbol{R},\boldsymbol{t}}{\operatorname{argmin}} \sum_{(i,j)\,\in\, C} \|\boldsymbol{b}_j - \boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t}\|^2 . \tag{1}$$

A comparison of four algorithms for solving this problem can be found in [13]. We selected the algorithm based on singular value decomposition (SVD), which consistently provides a high level of accuracy, stability, and speed. First, the center of mass $\bar{\boldsymbol{a}}$ of the selected data points and the center of mass $\bar{\boldsymbol{b}}$ of the corresponding model points are computed according to

$$\bar{\boldsymbol{a}} = \frac{1}{|C|} \sum_{(i,j)\,\in\, C} \boldsymbol{a}_i \ , \quad \bar{\boldsymbol{b}} = \frac{1}{|C|} \sum_{(i,j)\,\in\, C} \boldsymbol{b}_j \ . \tag{2}$$

Centralizing the point sets yields the minimization problem

$$\boldsymbol{R}^* = \underset{\boldsymbol{R}}{\operatorname{argmin}} \sum_{(i,j)\,\in\, C} \left\| \left(\boldsymbol{b}_j - \bar{\boldsymbol{b}}\right) - \boldsymbol{R}\left(\boldsymbol{a}_i - \bar{\boldsymbol{a}}\right) \right\|^2 . \tag{3}$$

This problem is solved by computing the SVD of the matrix

$$\boldsymbol{K} = \sum_{(i,j)\,\in\, C} \left(\boldsymbol{b}_j - \bar{\boldsymbol{b}}\right) \left(\boldsymbol{a}_i - \bar{\boldsymbol{a}}\right)^T = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^T \tag{4}$$

and setting $\boldsymbol{R}^* = \boldsymbol{U}\boldsymbol{V}^T$. It is possible that the computed matrix $\boldsymbol{R}^*$ is not a pure rotation, but includes a reflection. This special case is easily corrected by multiplying the third column of matrix $\boldsymbol{U}$ by -1. Finally, the translation vector $\boldsymbol{t}^*$ can be computed as $\boldsymbol{t}^* = \bar{\boldsymbol{b}} - \boldsymbol{R}^* \bar{\boldsymbol{a}}$.

The standard ICP algorithm is stopped when the change of the registration error falls below a specified threshold. Due to the outlier rejection, which can cause a temporary increase of the registration error, it is better to monitor the change of the motion parameters in our algorithm. Additionally, a threshold for the maximum number of iterations prevents an infinite loop in the rare case of divergence.

3. INTEGRATED SCALE ESTIMATION

Accurate registration of two differently scaled point sets is very useful for applications that work with 3-D data generated by structure-from-motion algorithms. Our approach integrates the scale estimation into the ICP algorithm, so that rotation, translation, and scale factor are estimated simultaneously in every iteration of the algorithm. Consequently, in the same way that rotation and translation have to be roughly initialized, the scale factor also has to be roughly known.

With the integrated scale factor estimation, the minimization problem in (1) becomes

$$(\boldsymbol{R}^*, \boldsymbol{t}^*, s^*) = \underset{\boldsymbol{R},\boldsymbol{t},s}{\operatorname{argmin}} \sum_{(i,j)\,\in\, C} \|\boldsymbol{b}_j - s\boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t}\|^2 . \tag{5}$$

It is interesting to note that the estimation of the rotation matrix is not affected by the introduced scale factor at all. Applying any scale factor to the data point set amounts to multiplying matrix $\boldsymbol{K}$ in (4) by this scale factor, which does not change the matrices $\boldsymbol{U}$ and $\boldsymbol{V}^T$. Therefore, the rotation matrix can be computed first.

Now, the scale factor is the only unknown in the minimization problem

$$s^* = \underset{s}{\operatorname{argmin}} \sum_{(i,j)\,\in\, C} \left\| \left(\boldsymbol{b}_j - \bar{\boldsymbol{b}}\right) - s\boldsymbol{R}^* \left(\boldsymbol{a}_i - \bar{\boldsymbol{a}}\right) \right\|^2 . \tag{6}$$

After defining the vectors

$$\tilde{\boldsymbol{b}_j} = \left(\boldsymbol{b}_j - \bar{\boldsymbol{b}}\right) \ , \quad \tilde{\boldsymbol{a}_i} = \boldsymbol{R}^* \left(\boldsymbol{a}_i - \bar{\boldsymbol{a}}\right) \ , \tag{7}$$

the scale factor s^* is given by

$$s^* = \sum_{(i,j)\,\in\, C} \tilde{\boldsymbol{b}_j}^T \tilde{\boldsymbol{a}_i} \ \Big/ \sum_{(i,j)\,\in\, C} \tilde{\boldsymbol{a}_i}^T \tilde{\boldsymbol{a}_i} \ . \tag{8}$$

As the last step of the new algorithm, we compute the translation vector $\boldsymbol{t}^*$ according to $\boldsymbol{t}^* = \bar{\boldsymbol{b}} - s^* \boldsymbol{R}^* \bar{\boldsymbol{a}}$.

The additional operations required by the scale estimation in every iteration only slightly increase the computation time of the algorithm. The effects of the scale estimation on the number of iterations of the ICP algorithm will be evaluated experimentally in the next section.

4. EXPERIMENTAL EVALUATION

For the experimental evaluation of our ICP algorithm with integrated scale estimation, we used two point sets generated by structure-from-motion algorithms with 3000 points each. The first point set P_{head} was reconstructed from a human head, the second point set P_{desk} represents the 3-D structure of a cluttered desk. Both point sets were scaled so that they fit into a cube with an edge length of 100 units. An image from each video sequence used for reconstruction and an image of each point set can be seen in Fig. 1.

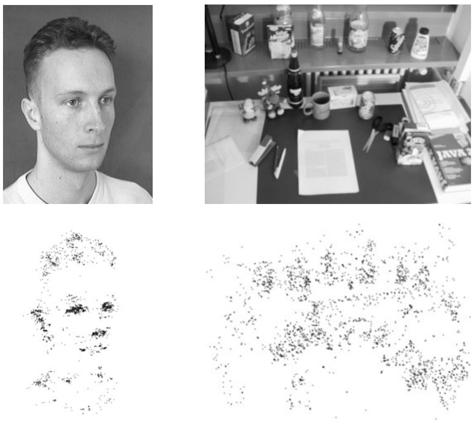

Figure 1: Point sets used for evaluation

The point sets themselves were used as model point sets. The creation of the data point sets started with copying the model point set. Then, Gaussian noise with a standard deviation of 0.2 was applied to every coordinate of every point. Finally, a specified rotation, translation, and scaling were applied to the data point set.

We evaluated the ICP algorithm with and without scale estimation. Different rotation angles, translation vector lengths, and scale factors were tested with 1000 trials for each setting. For each trial, the noise, the rotation axis, and the translation direction were determined randomly.

The basin of convergence of the algorithms was measured by observing the percentage of successful registrations. To this end, we computed the motion required to move the data point set from the estimated position to the correct position. A registration was considered successful when the required motion had a rotation angle of less than 0.1°, a translation of less than 0.025 units, and a scale factor between 0.999 and 1.001. For our configuration, these thresholds reliably separate correct registrations, which are markedly more accurate, from incorrect registrations, which are much further off.

In the first experiment, we tested the basin of convergence of the algorithms with respect to the rotation angle. The translation vector length was set to 7.5 units, the scale factor to 1.0. Fig. 2 illustrates that the registration percentage is close to 100 percent for rotation angles of up to 30°. For larger angles, the added scale estimation decreases the registration percentage moderately for the point set P_{head} and strongly for the point set P_{desk}.

In the second experiment, we evaluated the basin of convergence with respect to the translation vector length. The rotation angle was set to 15°, and the scale factor was set to 1.0. Fig. 3 shows that the results differ dramatically for both point sets. For the point set P_{head}, the registration percentage begins to drop at translations of about 10 units. For larger translations, the algorithm with scale estimation fails in notably more trials than the standard algorithm. For the point set P_{desk}, on the other hand, the standard algorithm is not affected by translations of up to 30 units, whereas the performance with scale estimation degrades rapidly for translations of more than 10 units.

When the translation is large, model points on the far side of the data point set are not chosen as corresponding points. Consequently, in the first iteration, the scale of the

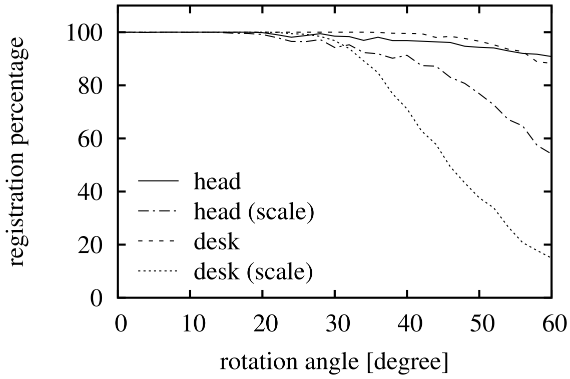

Figure 2: Evaluation of the basin of convergence of the ICP algorithm (standard / with scale estimation) with respect to the rotation angle

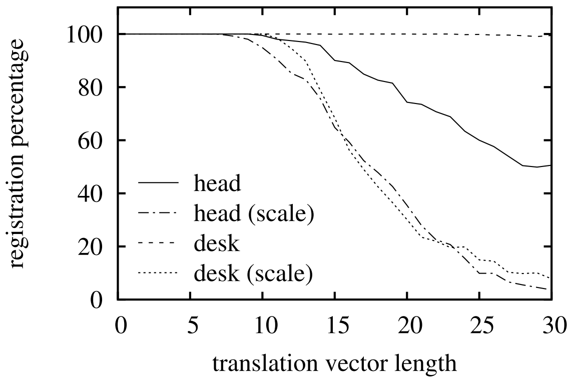

Figure 3: Evaluation of the basin of convergence of the ICP algorithm (standard / with scale estimation) with respect to the translation vector length

model point set is underestimated relative to the data point set, and the data point set is scaled down to match this scale before the second iteration. The effects of this behavior are further examined in the next experiment.

Finally, the basin of convergence with respect to the scale factor was tested in the third experiment. For this experiment, we used a rotation angle of 15° and a translation vector length of 7.5 units. Please note that the scale factor in Fig. 4 denotes the estimated scale factor, so that for a scale factor of 0.5, the data point set has twice the scale of the model point set. The registration percentage is close to 100 percent for scale factors ranging from 0.5 to 1.2. and rapidly drops for scale factors larger than 1.2.

When the data point set has a smaller scale than the model point set, it sometimes gets stuck inside the model point set. This happens because points at the outer edge of the model point set are never chosen as nearest neighbors of the data points, because other points inside the model point set are closer to the data points. Fig. 4 also demonstrates that the severity of this problem strongly depends on the shape of the point sets.

The computational efficiency of the integrated scale estimation is very good. The additional operations required in each iteration are negligible, because the nearest neighbor search requires most of the computation time. As the speed of convergence of the ICP algorithm is slightly slowed by the integrated scale estimation, the number of required it-

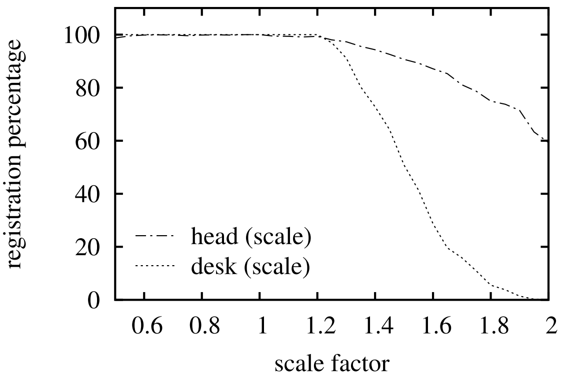

Figure 4: Evaluation of the basin of convergence of the ICP algorithm with scale estimation with respect to the scale factor

erations increases by approximately 25 percent. In our experiments, the run time for two point sets with 3000 points ranges from 60 ms to 200 ms on a Pentium 4 with 2.4 GHz.

Several conclusions can be drawn from our experiments. The basin of convergence of the ICP algorithm with and without integrated scale estimation depends strongly on the shape and structure of the point sets. The integrated scale estimation moderately decreases the basin of convergence of the ICP algorithm with respect to rotation and translation, but allows the accurate recovery of an unknown scale factor, which is not possible with the standard algorithm. Finally, we observed that our scale estimation better tolerates a larger scale of the data point set than a larger scale of the model point set.

5. CONCLUSION

We presented an algorithm for aligning two differently scaled 3-D point sets, which extends the ICP algorithm by estimating a scale factor between the point sets in every iteration. As the proposed motion estimation with integrated scale estimation is a drop-in replacement for the standard motion estimation, it can easily be applied to a wide range of existing variants of the ICP algorithm.

Our experiments proved that our algorithm can successfully estimate rotation, translation, and scale factor between roughly pre-aligned point sets. As the integrated estimation of the scale factor benefits from more accurate registration and outlier rejection in every iteration, it is more accurate than methods that estimate the scale factor before applying a standard ICP algorithm. Although the basin of convergence with respect to rotation and translation is moderately decreased by the integrated scale estimation, this can easily be compensated by more accurate pre-alignment or by starting the estimation from several different initial alignments.

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